

**GCE**

**Further Mathematics A**

**Y541/01: Pure Core 2**

Advanced GCE

**Mark Scheme for Autumn 2021**

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

© OCR 2021

## Annotations and abbreviations

| Annotation in RM assessor          | Meaning                                                                                             |
|------------------------------------|-----------------------------------------------------------------------------------------------------|
| ✓ and ✕                            |                                                                                                     |
| BOD                                | Benefit of doubt                                                                                    |
| FT                                 | Follow through                                                                                      |
| ISW                                | Ignore subsequent working                                                                           |
| M0, M1                             | Method mark awarded 0, 1                                                                            |
| A0, A1                             | Accuracy mark awarded 0, 1                                                                          |
| B0, B1                             | Independent mark awarded 0, 1                                                                       |
| SC                                 | Special case                                                                                        |
| ^                                  | Omission sign                                                                                       |
| MR                                 | Misread                                                                                             |
| BP                                 | Blank Page                                                                                          |
| NBOD                               | Benefit of doubt not given                                                                          |
| Highlighting                       |                                                                                                     |
|                                    |                                                                                                     |
| Other abbreviations in mark scheme | Meaning                                                                                             |
| dep*                               | Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark |
| cao                                | Correct answer only                                                                                 |
| oe                                 | Or equivalent                                                                                       |
| rot                                | Rounded or truncated                                                                                |
| soi                                | Seen or implied                                                                                     |
| www                                | Without wrong working                                                                               |
| AG                                 | Answer given                                                                                        |
| awrt                               | Anything which rounds to                                                                            |
| BC                                 | By Calculator                                                                                       |
| DR                                 | This question included the instruction: In this question you must show detailed reasoning.          |

| Question |  |  | Answer                                                                                                                                                                                       | Marks      | AO   | Guidance                                                                                                                                                    |                                                                                                       |
|----------|--|--|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------|------|-------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------|
| 1        |  |  | $\begin{pmatrix} -12 & -1 & 5 \\ -1 & -20 & 3 \\ 7-6a & 3a-2 & -4a-5 \end{pmatrix} \text{ or } \begin{pmatrix} -4a & -1 & 5 \\ 11-4a & -20 & 3 \\ -8-a & 7 & -17 \end{pmatrix} \text{ seen}$ | <b>M1</b>  | 1.1  | Either product <b>AB</b> or <b>BA</b> calculated (but not if assigned incorrectly).<br>Alternatively: equivalent correct useful entries calculated for both | Condone 3 errors or omissions<br><br>This mark can be implied by sight of a correct equation          |
|          |  |  | $-12 = -4a \text{ or } -1 = 11 - 4a \text{ or } 7 - 6a = -8 - a \text{ or } 3a - 2 = 7 \text{ or } -4a - 5 = -17$                                                                            | <b>M1</b>  | 1.1  | Finding matrix products both ways and equating entries usefully                                                                                             | This mark can be implied by sight of a correct equation even if other entries or equations are wrong. |
|          |  |  | $a = 3$                                                                                                                                                                                      | <b>A1</b>  | 2.2a |                                                                                                                                                             | Cannot be awarded if either <b>AB</b> or <b>BA</b> has more than 3 errors                             |
|          |  |  |                                                                                                                                                                                              | <b>[3]</b> |      |                                                                                                                                                             |                                                                                                       |

| Question |     |       | Answer                                                                                                                                                                                                                                                                                                          | Marks                                                         | AO                        | Guidance                                                                                                                            |                                                                                                                                                                         |
|----------|-----|-------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------|---------------------------|-------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 2        | (a) | (i)   | DR<br>$3z_1 + 4z_2 = 3(3 - 7i) + 4(2 + 4i) = 17 - 5i$                                                                                                                                                                                                                                                           | <b>B1</b><br><b>[1]</b>                                       | 1.1                       |                                                                                                                                     |                                                                                                                                                                         |
|          |     | (ii)  | DR<br>$z_1 z_2 = (3 - 7i)(2 + 4i) = 6 + 12i - 14i - 28(-1)$<br><br>$= 34 - 2i$                                                                                                                                                                                                                                  | <b>M1</b><br><br><b>A1</b><br><b>[2]</b>                      | 1.1<br><br>1.1            | Attempted expansion with $i^2 = -1$ used and at least 3 correctly expanded terms                                                    | $-28(-1)$ can be simply +28                                                                                                                                             |
|          |     | (iii) | DR<br>$\frac{z_1}{z_2} = \frac{3 - 7i}{2 + 4i} = \frac{3 - 7i}{2 + 4i} \times \frac{2 - 4i}{2 - 4i}$<br>$= \frac{6 - 12i - 14i - 28}{4 + 16} = \frac{-22 - 26i}{20} = -\frac{11}{10} - \frac{13}{10}i$                                                                                                          | <b>M1</b><br><br><b>A1</b><br><b>[2]</b>                      | 1.1<br><br>1.1            | Multiplying top and bottom by (real multiple of) conjugate of bottom<br><br>Must see some evidence of expansion                     | Allow $\frac{-11 - 13i}{10}$ or $-\frac{11 + 13i}{10}$                                                                                                                  |
|          | (b) |       | DR<br>$\sqrt{3^2 + (-7)^2}$ or $\tan^{-1}\left(\frac{-7}{3}\right)$<br><br>$ z_1  = \sqrt{58}$ or awrt 7.62 or $\arg z_1 = \text{awrt } -1.17$ or 5.12 rads<br>$z_1 = \sqrt{58}\text{cis}(-1.17)$ or $z_1 = \sqrt{58}e^{-1.17i}$ or<br>$z_1 = \sqrt{58}(\cos(-1.17) + i\sin(-1.17))$ or<br>$[\sqrt{58}, -1.17]$ | <b>M1</b><br><br><b>A1</b><br><br><b>A1</b><br><br><b>[3]</b> | 1.1<br><br>1.1<br><br>2.5 | Explicit working must be seen<br><br><br>Must be in correct form with $\sqrt{58}$ exact and could be awrt 5.12 instead of $-1.17$ . | Other trig calculations could be sufficient for M1 provided that these are being used to find the argument.<br><br>Do not condone degrees<br><br>Condone round brackets |

| Question |     | Answer                                                                                                                                                                                                                                                                                                                                                                                                                                                 | Marks                                                                         | AO                                        | Guidance                                                                                                                                                  |                                                                                                                                                                |
|----------|-----|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------|-------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 3        | (a) | $\left( \begin{pmatrix} 1 \\ -3 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 2 \\ -2 \end{pmatrix} \right) \cdot \begin{pmatrix} 2 \\ -5 \\ -3 \end{pmatrix} = 4$ $2 + 15 - 9 + \lambda(6 - 10 + 6) = 4$ $8 + 2\lambda = 4 \Rightarrow 2\lambda = -4 \Rightarrow \lambda = -2 \text{ so}$ $\mathbf{r} = \begin{pmatrix} 1 \\ -3 \\ 3 \end{pmatrix} + -2 \begin{pmatrix} 3 \\ 2 \\ -2 \end{pmatrix} = \begin{pmatrix} -5 \\ -7 \\ 7 \end{pmatrix}$ | <b>M1</b><br><br><br><br><br><b>M1</b><br><br><b>A1</b><br><br><b>[3]</b>     | 1.1<br><br><br><br><br>1.1<br><br>1.1     | Substituting the expression for a point on the line into the equation of the plane<br><br><br><br><br>Dotting out to form and solve equation in $\lambda$ | <br><br><br><br><br><br><br>Condone coordinates                                                                                                                |
|          | (b) | $\frac{\begin{pmatrix} 2 \\ -5 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 2 \\ -2 \end{pmatrix}}{\sqrt{4+25+9}\sqrt{9+4+4}} \text{ soi}$ $= \frac{6-10+6}{\sqrt{38}\sqrt{17}} = \frac{2}{\sqrt{646}} = 0.07868...$ $\theta = \text{awrt } 85.5...^\circ \text{ soi}$ $(\phi = 90^\circ - 85.48...^\circ =) \text{ awrt } 4.51^\circ$                                                                                                               | <b>M1</b><br><br><br><br><br><br><br><b>A1</b><br><br><b>A1</b><br><b>[3]</b> | 1.1<br><br><br><br><br><br>1.1<br><br>1.1 | BC. Using $\cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{ \mathbf{a}  \mathbf{b} }$<br><br><br><br><br>Can be implied by correct final answer          | May see $\sin \phi = \frac{\mathbf{a} \cdot \mathbf{b}}{ \mathbf{a}  \mathbf{b} }$<br>Or use of cross product<br><br><br>or 1.49... rads<br><br>or 0.0788 rads |

| Question |     | Answer                                                                                                                                                                                                                                                                                                                                                                                                               | Marks                                                       | AO                                                                              | Guidance                                                                                                                                           |  |
|----------|-----|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------|---------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------|--|
| 3        | (c) | $\lambda = 1 \Rightarrow \mathbf{r} = \begin{pmatrix} 4 \\ -1 \\ 1 \end{pmatrix}$ $\mathbf{b} = \begin{pmatrix} 3 \\ 2 \\ -2 \end{pmatrix} \times \begin{pmatrix} 2 \\ -5 \\ -3 \end{pmatrix} = \begin{pmatrix} -16 \\ 5 \\ -19 \end{pmatrix}$ <p>So equation of <math>l_2</math> is <math>\mathbf{r} = \begin{pmatrix} 4 \\ -1 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} -16 \\ 5 \\ -19 \end{pmatrix}</math> oe</p> | <b>B1</b><br><br><br><br><br><br><br><br><br><br><b>[3]</b> | 3.1a<br><br><br><br><br><br><br><br><br><br>1.1                                 | Method shown or at least two terms correctly evaluated<br><br><br><br><br><br><br><br><br><br>Must be $\mathbf{r} =$ . Allow parameter $\lambda$ . |  |
| 4        |     | DR<br>$\sum_{r=1}^{100} (2r+3)^2 = 4 \sum_{r=1}^{100} r^2 + 12 \sum_{r=1}^{100} r + 9 \sum_{r=1}^{100} 1$ $\sum_{r=1}^{100} r^2 = \frac{1}{6} \times 100(100+1)(2 \times 100 + 1)$ $4 \times 338350 + 12 \times \frac{1}{2} \times 100 \times 101 + 900 = 1414900$                                                                                                                                                   | <b>B1</b><br><br><br><br><br><br><br><br><br><br><b>[3]</b> | 3.1a<br><br><br><br><br><br><br><br><br><br>1.1a<br><br><br><br><br><br><br>1.1 | Expanding and separating<br><br><br><br><br><br><br><br><br><br>Use of formula for $\sum_{r=1}^{100} r^2$                                          |  |

| Question |     |  | Answer                                                                                                                                                                                                                                                                         | Marks                                                                  | AO                           | Guidance                                                                                                                                                                      |                                                                                                                                                                |
|----------|-----|--|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------|------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 5        | (a) |  | DR<br>$\text{RHS} = 2\cosh^2 x - 1 = 2\left(\frac{e^x + e^{-x}}{2}\right)^2 - 1$ $= 2\left(\frac{e^{2x} + 2 + e^{-2x}}{4}\right) - 1 = \frac{e^{2x} + 2 + e^{-2x}}{2}$ $= \frac{e^{2x} + e^{-2x}}{2} = \cosh 2x = \text{LHS}$                                                  | <b>M1</b>                                                              | 2.1                          | Uses correct exponential form in an attempt at proof                                                                                                                          |                                                                                                                                                                |
|          |     |  |                                                                                                                                                                                                                                                                                | <b>A1</b><br><b>[2]</b>                                                | 2.1                          | AG                                                                                                                                                                            | Proof must be complete                                                                                                                                         |
|          | (b) |  | DR<br>$2\cosh^2 x - 1 = 3\cosh x + 1$ $\Rightarrow 2\cosh^2 x - 3\cosh x - 2 = 0$ $(2\cosh x + 1)(\cosh x - 2) = 0$<br>$\cosh x = 2 \text{ or } -\frac{1}{2}$ $\cosh x \geq 1 \text{ so } \neq -\frac{1}{2}$<br>$x = \cosh^{-1} 2 = \ln(2 + \sqrt{3})$ $x = \ln(2 - \sqrt{3})$ | <b>M1</b><br><br><b>M1</b>                                             | 3.1a<br>1.1                  | Use of identity in (a) to leave a three term quadratic equation in just coshx<br>Attempt to solve eg<br>$\frac{-(-3) \pm \sqrt{(-3)^2 - 4 \times 2 \times (-2)}}{2 \times 2}$ | $\text{or } 2\left(\cosh x - \frac{3}{4}\right)^2 - \frac{9}{8} - 2 = 0$<br>Or solves quadratic BC<br><br><br>Or $x = -\ln(2 + \sqrt{3})$<br>Mark final answer |
|          |     |  |                                                                                                                                                                                                                                                                                | <b>A1</b><br><b>A1</b><br><br><b>A1</b><br><br><b>A1</b><br><b>[6]</b> | 1.1<br>2.3<br><br>1.1<br>1.1 | Justification must be seen and must contain no incorrect statements<br>For either correct answer seen<br><br>Both correct values for x                                        |                                                                                                                                                                |

|   |     |  | Answer                                                                                                                                                                                                                                                                                                                                                                        | Marks                                                                   | AO                                    | Guidance                                                                                                  |                                    |
|---|-----|--|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------|---------------------------------------|-----------------------------------------------------------------------------------------------------------|------------------------------------|
| 6 | (a) |  | DR<br>A shear which leaves the $x$ -axis invariant and which transforms the point $(0, 1)$ to the point $(2, 1)$ .                                                                                                                                                                                                                                                            | <b>B1</b><br><br>[1]                                                    | 2.2a                                  | Or any useful point transformed to its image                                                              | not “scale factor” or sf           |
|   | (b) |  | DR<br>$\det A = 1 \times 1 - 0 \times 2 = 1$ and this is the area scale factor                                                                                                                                                                                                                                                                                                | <b>B1</b><br><br>[1]                                                    | 2.4                                   | Both                                                                                                      | Detailed calculation must be shown |
|   | (c) |  | DR<br>$\begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}$ seen<br>$\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} 7 & 2 \\ 3 & 1 \end{pmatrix}$<br>$\begin{pmatrix} 1 & 0 \\ 0 & p \end{pmatrix} \begin{pmatrix} 7 & 2 \\ 3 & 1 \end{pmatrix}$<br>$= \begin{pmatrix} 7 & 2 \\ 3p & p \end{pmatrix} \Rightarrow p = 7$ | <b>B1</b><br><br><b>B1</b><br><br><b>M1</b><br><br><b>A1</b><br><br>[4] | 3.1a<br><br>1.1<br><br>1.1<br><br>1.1 | BC<br><br>Correct form for stretch multiplied into their matrix in either order<br>Correct multiplication |                                    |

| Question |     | Answer                                                                                                                                                                                                                                                                                                                                         | Marks                                                                                                   | AO                                                       | Guidance                                                                                                                                                                                                                             |                                                                                                                                                                                        |
|----------|-----|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------|----------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 7        | (a) | DR<br>$\frac{x^3 + x^2 + 9x - 1}{x^3 + x^2 + 4x + 4} \equiv \frac{x^3 + x^2 + 4x + 4 + 5x - 5}{x^3 + x^2 + 4x + 4}$ $= 1 + \frac{5x - 5}{x^3 + x^2 + 4x + 4}$ So $A = 1$ , $B = 5$ and $C = -5$                                                                                                                                                | <b>B1</b><br><br><b>[1]</b>                                                                             | 3.1a                                                     | Attempt to divide out improper fraction. Could be by symbolic division or other valid method (eg comparing coefficients or substitution of values for $x$ )                                                                          | Allow embedded answers                                                                                                                                                                 |
|          | (b) | DR<br>$x^3 + x^2 + 4x + 4 = (x+1)(x^2 + 4)$ $\frac{5x - 5}{x^3 + x^2 + 4x + 4} = \frac{D}{x+1} + \frac{Ex + F}{x^2 + 4}$ $D(x^2 + 4) + (x+1)(Ex + F) = 5x - 5$ $x = -1 \Rightarrow 5D = -10 \Rightarrow D = -2$ $x = 0 \Rightarrow -2 - F = -5 \Rightarrow F = 3$ $x^2 : D + E = 0 \Rightarrow E = 2$ $1 - \frac{2}{x+1} + \frac{2x+3}{x^2+4}$ | <b>B1</b><br><br><b>M1</b><br><br><br><br><b>A1</b><br><br><b>A1</b><br><br><b>A1</b><br><br><b>[5]</b> | 3.1a<br><br>1.2<br><br><br><br>1.1<br><br>1.1<br><br>1.1 | Correct factorisation of cubic seen in working<br>Correct form for partial fractions equated to their remainder rational fraction from (a). Follow through their division and factorisation.<br>Or equivalent to find $D$ correctly. | Could be from improper fraction<br><br><br><br><br><br><br>Allow ft A1 for second and third coefficients found.<br><br><br>Or $1 - \frac{2}{x+1} + \frac{2x}{x^2+4} + \frac{3}{x^2+4}$ |

| Question |     | Answer                                                                                                                                                                                                                                                                                                                                                                  | Marks                                                                                               | AO                                                    | Guidance                                                                                                                                                                                                                                                                                                                                          |  |
|----------|-----|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------|-------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|
| 7        | (c) | <p>DR</p> $\int_0^2 \frac{x^3 + x^2 + 9x - 1}{x^3 + x^2 + 4x + 4} dx = \int_0^2 1 - \frac{2}{x+1} + \frac{2x}{x^2+4} + \frac{3}{x^2+4} dx$<br>$= \left[ x - 2 \ln(x+1) + \ln(x^2+4) + \frac{3}{2} \tan^{-1}\left(\frac{x}{2}\right) \right]_0^2$<br>$\left( 2 - 2 \ln 3 + \ln 8 + \frac{3\pi}{8} \right) - \ln 4$<br>$2 + \ln\left(\frac{2}{9}\right) + \frac{3}{8}\pi$ | <p><b>*M1</b></p><br><p><b>dep*M1</b></p><br><p><b>M1</b></p><br><p><b>A1</b></p> <p><b>[4]</b></p> | <p>3.1a</p><br><p>1.1</p><br><p>1.1</p><br><p>1.1</p> | <p>Split term with <math>x^2 + 4</math> in denominator and <math>ax + b</math> in numerator</p><br><p>Correctly integrate <i>their</i> expression (ignore limits)</p><br><p>Correctly substitute limits to produce exact values and evaluate their <math>\tan^{-1}</math> term</p><br><p><math>a = 2, b = \frac{2}{9}, c = \frac{3}{8}</math></p> |  |

|   |     |                                                                                                                          |        |      |                                                                                  |                                              |
|---|-----|--------------------------------------------------------------------------------------------------------------------------|--------|------|----------------------------------------------------------------------------------|----------------------------------------------|
| 8 | (a) | $F = ma = 2 \frac{dv}{dt} = 4e^{-2t} - kv$                                                                               | M1     | 3.3  | Use of NII with $m$ and $a$ replaced and with 2 forces, the given force and $kv$ | $F=ma$ can be implicit here                  |
|   |     | $t = \ln 2, v = 0.5, F = 0 \Rightarrow 0 = 1 - 0.5k$                                                                     | M1     | 2.2a | Use of given conditions to derive an equation in $k$                             | Can be done first                            |
|   |     | $k = 2 \Rightarrow 2 \frac{dv}{dt} = 4e^{-2t} - 2v \Rightarrow \frac{dv}{dt} + v = 2e^{-2t}$                             | A1     | 1.1  | AG                                                                               | Complete argument including $F=ma$           |
|   |     |                                                                                                                          | [3]    |      |                                                                                  |                                              |
|   | (b) | IF = $e^{\int 1 dt} = e^t$                                                                                               | *B1    | 1.1  |                                                                                  | Or CF                                        |
|   |     | $e^t \frac{dv}{dt} + e^t v = \frac{d}{dt}(e^t v) = e^t \times 2e^{-2t}$                                                  | *M1    | 1.1  | Multiplying by IF and writing LHS as an exact derivative                         | Or subst correct PI into DE                  |
|   |     | $e^t v = \int 2e^{-t} dt = -2e^{-t} + c$                                                                                 | A1     | 1.1  | “+ $c$ ” required                                                                | Or GS $v = Ae^{-t} - 2e^{-2t}$               |
|   |     | $t = 0, v = 0 \Rightarrow c = 2$                                                                                         | dep*M1 | 3.4  | Use of initial conditions to derive a value for $c$                              | Or using alternative boundary condition      |
|   |     | $v = 2e^{-t} - 2e^{-2t}$                                                                                                 | A1     | 3.4  |                                                                                  |                                              |
|   | (c) |                                                                                                                          | [5]    |      |                                                                                  |                                              |
|   |     | As $t \rightarrow \infty, v \rightarrow 0$                                                                               | M1     | 3.4  |                                                                                  |                                              |
|   |     | So speed starts at 0 and ends at 0 (and is continuous and positive between) so must reach a maximum somewhere in $t > 0$ | A1     | 2.4  |                                                                                  |                                              |
|   | (d) |                                                                                                                          | [2]    |      |                                                                                  |                                              |
|   |     | $v$ is max when $\frac{dv}{dt} = 0$ so $t = \ln 2$                                                                       | M1     | 2.2a | Deducing time when $v$ is maximum                                                | Or by finding expression for $\frac{dv}{dt}$ |
|   |     | So $v_{\max} = 0.5$ (given) (or<br>$v_{\max} = 2e^{-\ln 2} - 2e^{-2\ln 2} = 1 - \frac{2}{4} = \frac{1}{2}$ )             | A1     | 3.4  |                                                                                  | and solving $\frac{dv}{dt} = 0$              |
|   |     |                                                                                                                          | [2]    |      |                                                                                  |                                              |

| Question |     | Answer                                                                                                                                                                                                                                                                                                                                           | Marks                                                        | AO                                    | Guidance                                                                                                                                                                                                                                                                      |                                                                       |
|----------|-----|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------|---------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------|
| 8        | (e) | $v = \frac{dx}{dt} = 2e^{-t} - 2e^{-2t} \Rightarrow x = -2e^{-t} + e^{-2t} + d$ $t = 0, x = 0 \Rightarrow 0 = -2 + 1 + d \Rightarrow d = 1$ $0.9 = -2e^{-t} + e^{-2t} + 1$ $(e^{-t})^2 - 2e^{-t} + 0.1 = 0 \Rightarrow e^{-t} = \frac{10 \pm 3\sqrt{10}}{10}$ $\Rightarrow t = \ln\left(\frac{10}{10 - 3\sqrt{10}}\right) = 2.97 \text{ (3 sf)}$ | <b>M1</b><br><br><b>M1</b><br><br><b>M1</b><br><br><b>A1</b> | 3.3<br><br>3.3<br><br>3.5a<br><br>2.3 | Integrating to find expression for $x$<br><br>Using initial conditions to find value of (new) constant<br>Recognising that the model is only valid when $x$ lies between 0 and 0.9<br><br>Rejecting $t = \ln\left(\frac{10}{10 + 3\sqrt{10}}\right) < 0$<br>(can be implicit) | Or definite integral with correct lower limit..<br>...and upper limit |
| 9        | (a) | $\mathbf{A}^2 = \begin{pmatrix} 4 & 12 \\ 0 & 4 \end{pmatrix}, \mathbf{A}^3 = \begin{pmatrix} 8 & 36 \\ 0 & 8 \end{pmatrix},$ $\mathbf{A}^4 = \begin{pmatrix} 16 & 96 \\ 0 & 16 \end{pmatrix}$ Conjecture: $\mathbf{A}^n = \begin{pmatrix} 2^n & 3n \times 2^{n-1} \\ 0 & 2^n \end{pmatrix}$                                                     | <br><br><b>B1</b><br><br><b>B1</b>                           | <br><br>2.2a<br><br>2.2b              | BC<br><br>Allow this mark for any conjecture which works for $n = 1, 2, 3$ and 4.                                                                                                                                                                                             |                                                                       |

| Question |     | Answer                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | Marks                                                               | AO                                           | Guidance                                                                                                                                                                                                                                                                                                                                                                                                                                                               |                                                                |
|----------|-----|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------|----------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------|
| 9        | (b) | <p>Basis case: <math>n = 1</math>:</p> $\mathbf{A}^1 = \begin{pmatrix} 2^1 & 3 \times 1 \times 2^0 \\ 0 & 2^1 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 0 & 2 \end{pmatrix} = \mathbf{A} \text{ so true}$ <p>for <math>n = 1</math></p> <p>Assume true for <math>n = k</math></p> <p>ie <math>\mathbf{A}^k = \begin{pmatrix} 2^k &amp; 3k \times 2^{k-1} \\ 0 &amp; 2^k \end{pmatrix}</math></p> $\mathbf{A}^{k+1} = \mathbf{A}^k \mathbf{A} = \begin{pmatrix} 2^k & 3k \times 2^{k-1} \\ 0 & 2^k \end{pmatrix} \begin{pmatrix} 2 & 3 \\ 0 & 2 \end{pmatrix}$ $= \begin{pmatrix} 2^{k+1} & 3 \times 2^k + 3k \times 2^k \\ 0 & 2^{k+1} \end{pmatrix}$ $= \begin{pmatrix} 2^{k+1} & 3(k+1) \times 2^k \\ 0 & 2^{k+1} \end{pmatrix}$ <p>So true for <math>n = k \Rightarrow</math> true for <math>n = k + 1</math>. But true for <math>n = 1</math>.</p> <p>So true for all positive integer <math>n</math></p> | <p><b>B1</b></p> <p><b>M1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> | <p>2.1</p> <p>2.1</p> <p>2.2a</p> <p>2.4</p> | <p>Allow this mark even if the conjecture is wrong, provided that it works for <math>n = 1</math></p> <p>Must have statement in terms of some other variable than <math>n</math>. Conjecture need not be correct.</p> <p>Uses inductive hypothesis properly &amp; expands</p> <p>AG. Manipulating terms correctly and convincingly to obtain required form. Some intermediate working must be seen and a clear conclusion must be given for the induction process.</p> | <p>A formal proof by induction is required for full marks.</p> |
|          |     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | [4]                                                                 |                                              |                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |                                                                |

| Question |     |  | Answer                                                                                                                                                                                                                 | Marks                                             | AO   | Guidance                                                        |  |
|----------|-----|--|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------|------|-----------------------------------------------------------------|--|
| 10       | (a) |  | DR<br>$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots = (1+x) + \left( \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \right)$ $x > 0 \Rightarrow \frac{x^2}{2!} + \frac{x^3}{3!} + \dots > 0$ $\Rightarrow e^x > 1+x$ | <b>M1</b>                                         | 1.1  | Quoting and <i>using</i> the Maclaurin series                   |  |
|          |     |  |                                                                                                                                                                                                                        | <b>A1</b><br><b>[2]</b>                           | 2.2a | AG. Result with sufficient justification                        |  |
|          | (b) |  | DR<br>$t = x+1 \Rightarrow e^{t-1} > t \Rightarrow \frac{e^t}{e} > t \Rightarrow e^t > et$                                                                                                                             | <b>B1</b><br><b>[1]</b>                           | 3.1a | AG                                                              |  |
|          | (c) |  | DR<br>$t = \frac{\pi}{e} > 1 \text{ since } 2 < e < 3 \text{ and } \pi > 3$ $e^{\frac{\pi}{e}} > e \times \frac{\pi}{e} (= \pi)$ $\Rightarrow e^\pi > \pi^e \text{ (ie } e^\pi \text{ is greater)}$                    | <b>B1</b>                                         | 3.1a | Some justification that $t > 1$ is required                     |  |
|          |     |  |                                                                                                                                                                                                                        | <b>M1</b>                                         | 3.1a | Substituting their choice into the inequality                   |  |
|          |     |  |                                                                                                                                                                                                                        | <b>A1</b>                                         | 1.1  | Answer without use of inequality in part (b) scores <b>M0A0</b> |  |
|          |     |  | <b>Alternative method</b><br>$t = \ln \pi$ $e^{\ln \pi} > e \ln \pi$ $e^{\ln \pi} > e \ln \pi$ $\pi > \ln(\pi^e)$ $e^\pi > \pi^e$                                                                                      | <b>B1</b><br><b>M1</b><br><b>A1</b><br><b>[3]</b> |      | Some justification that $t > 1$ is required                     |  |

**OCR (Oxford Cambridge and RSA Examinations)**  
**The Triangle Building**  
**Shaftesbury Road**  
**Cambridge**  
**CB2 8EA**

**OCR Customer Contact Centre**

**Education and Learning**

Telephone: 01223 553998

Facsimile: 01223 552627

Email: [general.qualifications@ocr.org.uk](mailto:general.qualifications@ocr.org.uk)

[www.ocr.org.uk](http://www.ocr.org.uk)

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored